

A note on spherically symmetric, static spacetimes in Kanno-Soda on-brane gravity

Sayan Kar[#], Sayantani Lahiri^{†,‡} and Soumitra SenGupta ^{**}

[#]*Department of Physics and Center for Theoretical Studies*

Indian Institute of Technology, Kharagpur, 721 302, India,

[†]*Institute for Physics, University Oldenburg, D-26111 Oldenburg, Germany,*

[‡]*ZARM, University of Bremen, Am Fallturm, 28359 Bremen, Germany and*

^{**}*Department of Theoretical Physics,*

Indian Association for the Cultivation of Science

2A and 2B Raja S.C. Mallick Road, Jadavpur, Kolkata 700 032, India.

Abstract

Spherically symmetric, static on-brane geometries in the Kanno-Soda (KS) effective scalar-tensor theory of on-brane gravity are discussed. In order to avoid brane collisions and/or an infinite inter-brane distance, at finite values of the brane coordinates, it is necessary that the radion scalar be everywhere finite and non-zero. This requirement constrains the viability of the standard, well-known solutions in General Relativity (GR), in the context of the KS effective theory. The radion for the Schwarzschild solution does not satisfy the above requirement. For the Reissner–Nordstrom (RN) naked singularity and the extremal RN solution, one can obtain everywhere finite, non-zero radion profiles, though the required on-brane matter violates the Weak Energy Condition. In contrast, for the RN black hole, the radion profile yields a divergent inter-brane distance at the horizon, which makes the solution unphysical. Thus, both the Schwarzschild and the RN solutions can be meaningful in the KS effective theory, only in the trivial GR limit, i.e. with a constant, non-zero radion.

*Electronic address: sayan@iitkgp.ac.in, sayantani.lahiri@gmail.com, tpssg@iacs.res.in

I. INTRODUCTION

Effective, on-brane theories of gravity have been in vogue ever since the Randall-Sundrum warped braneworld scenario was proposed[1]. Among such four dimensional gravity theories, the most well-known one is due to Shiromizu, Maeda and Sasaki (SMS) [2] which considers a bulk with an infinite extra-dimension and a single brane. There have been proposals on effective theories in a two-brane set-up. In this article, we consider one such effective theory due to Kanno and Soda (KS) [3]. A major difference between the SMS and the KS effective theories is the presence of a non-local (bulk dependent) term in the former and the absence of any non-locality in the latter. Our objective is to look for solutions in the KS effective theory, keeping in mind that the radion field, which is linked to the distance between the branes, (i) is never zero in value (thus, avoiding brane collisions) and (ii) does not diverge at any finite value of the brane coordinates.

Recently [4], we have discussed some cosmological and spherically symmetric, static spacetimes in this four dimensional, effective, on-brane, scalar-tensor theory of gravity. The spherically symmetric, static solutions obtained in [4] turned out to be the Majumdar-Papapetrou solution [5] with the source being the effective scalar field (radion) energy-momentum and additional on-brane matter. Here, we ask a broader question: are the standard General Relativity (GR) solutions like the Schwarzschild or the Reissner-Nordstrom (RN) permissible in the KS theory? Obviously, we do not expect the GR solutions to arise in the KS theory with the same matter content as in GR. Rather, we would like to find out if the radion energy momentum and extra on-brane matter can conspire in unison to allow the Schwarzschild or the RN solution in the KS effective theory.

It should be noted that there are several aspects related to this question, a couple of which we have already mentioned. In addition to having a radion which is finite and non-zero everywhere, the on-brane matter must also be physically reasonable in the classical sense, i.e. it must satisfy one of the well-known energy conditions, such as the Weak Energy Condition or the Null Energy Condition [7].

We may recall that the Reissner-Nordstrom solution does arise as a solution [6] of the Shiromizu-Maeda-Sasaki (SMS) single brane effective Einstein equations [2], where the non-local contribution from the bulk Weyl tensor (the traceless $\mathcal{E}_{\mu\nu}$) acts as its source without any explicit on-brane matter. The functional form of the $\mathcal{E}_{\mu\nu}$ depends on the bulk Weyl

tensor and other features of the bulk geometry. It cannot be determined uniquely from the knowledge of four dimensional, on-brane physics. In contrast, in the KS effective theory, the influence of the bulk is exclusively through the radion field which depends only on the brane coordinates. It is therefore meaningful to ask whether the equations which arise in the effective, on-brane Kanno-Soda theory (which are local and different from those obtained in the single brane SMS effective theory) also admit a RN or a Schwarzschild solution, in some way.

We will mainly work with the Reissner–Nordstrom solution written in isotropic coordinates. After obtaining the radion profiles in the various cases, we will see if the radion satisfies the necessary requirements. Subsequently, we will analyse the nature of required on-brane matter with reference to the Weak Energy Condition [7].

II. THE KANNO-SODA EFFECTIVE THEORY: MAIN EQUATIONS

The effective on-brane scalar-tensor theories developed by Kanno and Soda [3] in the context of the Randall–Sundrum two-brane model leads to the following Einstein-like equations on the visible ‘b’ brane [3],

$$G_{\mu\nu} = \frac{\kappa^2}{l\Phi} T_{\mu\nu}^b + \frac{\kappa^2(1+\Phi)}{l\Phi} T_{\mu\nu}^a + \frac{1}{\Phi} (\nabla_\mu \nabla_\nu \Phi - g_{\mu\nu} \nabla^\alpha \nabla_\alpha \Phi) - \frac{3}{2\Phi(1+\Phi)} \left(\nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} g_{\mu\nu} \nabla^\alpha \Phi \nabla_\alpha \Phi \right) \quad (1)$$

Here $g_{\mu\nu}$ is the on-brane metric, the covariant differentiation is defined with respect to $g_{\mu\nu}$ and we have taken the five dimensional line element as,

$$ds_5^2 = e^{2\phi(x)} dy^2 + \tilde{g}_{\mu\nu}(y, x^\mu) dx^\mu dx^\nu \quad (2)$$

κ^2 is the $5D$ gravitational coupling constant. $T_{\mu\nu}^a$, $T_{\mu\nu}^b$ are the stress-energy on the Planck brane and the visible brane respectively. The appearance of $T_{\mu\nu}^a$ (matter energy momentum on the ‘a’ brane) in the field equations on the ‘b’ brane, inspired the usage of the term ‘quasi-scalar-tensor theory’. However, if we assume $T_{\mu\nu}^a = 0$ then we have the usual scalar-tensor theory.

We denote $d(x)$ as the proper distance between branes located at $y = 0$ and $y = l$. $d(x)$ is defined as,

$$d(x) = \int_0^l e^{\phi(x)} dy \quad (3)$$

We further define $\Phi = e^{2\frac{d}{l}} - 1$. It may be observed that the viability of such a model with a everywhere finite and non-zero brane separation, implies that (i) the minimum of $d(x)$ is not equal to zero and (ii) $d(x)$ is never infinity at any finite value of the brane coordinates. This, in turn indicates that the value of $\Phi(x)$ is always greater than zero and $\Phi(x)$ never becomes infinity.

Note that the radion contribution on the R. H. S. of the field equation is traceless, which is reminiscent of the traceless $\mathcal{E}_{\mu\nu}$ in the SMS effective theory.

The scalar field equation of motion on the visible brane is given as,

$$\nabla^\alpha \nabla_\alpha \Phi = \frac{\kappa^2}{l} \frac{T^a + T^b}{2\omega + 3} - \frac{1}{2\omega + 3} \frac{d\omega}{d\Phi} (\nabla^\alpha \Phi) (\nabla_\alpha \Phi) \quad (4)$$

where T^a , T^b are the traces of energy momentum tensors on Planck ('a') and visible ('b') branes, respectively. The coupling function $\omega(\Phi)$ expressed in terms of Φ is,

$$\omega(\Phi) = -\frac{3\Phi}{2(1 + \Phi)} \quad (5)$$

We know that gravity on both the branes are not independent. Dynamics on the Planck brane at $y = 0$ is linked to that on the visible brane through the relation [3] :

$$\Phi(x) = \frac{\Psi}{1 - \Psi} \quad (6)$$

where Ψ is the radion field as defined on Planck brane. The induced metric on the visible brane can be expressed in terms of Ψ as,

$$g_{\mu\nu}^{b-brane} = (1 - \Psi) [h_{\mu\nu} + g_{\mu\nu}^{(1)}(h_{\mu\nu}, \Psi, T_{\mu\nu}^a, T_{\mu\nu}^b, y = l)] \quad (7)$$

where $g_{\mu\nu}^{(1)}$ is the first order correction term (see [3] for details). It is possible to work with the gravity theory and the Ψ field equation on the 'a' brane.

In our work here, we assume that the on-brane stress energy is nonzero only on the 'b' brane (visible brane). We also assume that the on-brane matter is traceless and therefore, since the effective radion stress energy is also traceless, the Ricci scalar of the spacetime geometry is identically zero. Conversely, if we assume $R = 0$, the on-brane matter is traceless. This choice of $R = 0$ enables us to propose the standard General Relativity solutions (like Schwarzschild and Reissner-Nordstrom) as solutions in the Kanno-Soda effective theory with on-brane matter. The two main hurdles we need to address are therefore:

- Are the standard GR solutions also solutions in the effective theory, with a non-zero, everywhere finite radion?

- What is the nature of the on-brane matter required to support such standard GR solutions?

III. SPHERICALLY SYMMETRIC, STATIC SOLUTIONS

A. Line element, field equations and the radion

Let us assume a four dimensional line element on the visible brane, in isotropic coordinates, given as

$$ds^2 = -\frac{f^2(r)}{U^2(r)}dt^2 + U^2(r) [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \quad (8)$$

where $U(r)$ and $f(r)$ are the unknown functions to be determined from the Einstein-like equations. Using the above line element ansatz and the assumption that Φ is a function of r alone, we get the following field equations from the Einstein field equations mentioned above.

$$-2\frac{U''}{U} + \left(\frac{U'}{U}\right)^2 - 4\frac{U'}{Ur} = -\frac{\Phi'^2}{4\Phi(1+\Phi)} + \left(\frac{U'}{U} - \frac{f'}{f}\right)\frac{\Phi'}{\Phi} + \frac{\kappa^2}{l\Phi}\rho \quad (9)$$

$$-\left(\frac{U'}{U}\right)^2 + 2\frac{f'}{f}\left(\frac{U'}{U} + \frac{1}{r}\right) = -\frac{3\Phi'^2}{4\Phi(1+\Phi)} - \frac{U'\Phi'}{U\Phi} - \frac{2\Phi'}{\Phi r} - \frac{f'\Phi'}{f\Phi} + \frac{\kappa^2}{l\Phi}\tau \quad (10)$$

$$\left(\frac{U'}{U}\right)^2 + \frac{f''}{f} - 2\frac{f'}{f}\frac{U'}{U} + \frac{f'}{f}\frac{1}{r} = \frac{\Phi'^2}{4\Phi(1+\Phi)} + \frac{U'\Phi'}{U\Phi} + \frac{\Phi'}{\Phi r} + \frac{\kappa^2}{l\Phi}p \quad (11)$$

where ρ , τ and p correspond to on-brane matter and using the tracelessness condition we have $-\rho + \tau + 2p = 0$. We have absorbed a factor of U^2 in the definitions of ρ , τ and p .

On the other hand, the scalar (Φ) field equation gives

$$\Phi'' + \frac{f'}{f}\Phi' + 2\frac{\Phi'}{r} = \frac{\Phi'^2}{2(1+\Phi)} \quad (12)$$

which can be integrated once to get

$$\frac{\Phi'}{\sqrt{1+\Phi}} = \frac{2C_1}{r^2 f} \quad (13)$$

where C_1 is a positive, non-zero constant. It is useful to note here that the radion depends only on the metric function $f(r)$ and not on $U(r)$. We also know that the existence of a horizon in a static, spherically symmetric geometry is linked with the existence of zeros

in $f(r)$. Thus, Φ' will always diverge at the horizon of any spherically symmetric, static spacetime.

The field equations with the requirement of traceless on-brane matter, leads to the following equation for the metric function f and U :

$$\frac{U''}{U} + \frac{f''}{f} - \frac{f'}{f} \frac{U'}{U} + 2 \frac{f'}{fr} + 2 \frac{U'}{Ur} = 0 \quad (14)$$

A solution for f and U which satisfies the above tracelessness condition can be found by recalling the Reissner–Nordstrom solution written in isotropic coordinates. For such a solution we have,

$$f(r) = 1 - \frac{M^2}{4r^2} + \frac{e^2}{4r^2} \quad (15)$$

$$U(r) = 1 + \frac{M}{r} + \frac{M^2}{4r^2} - \frac{e^2}{4r^2} \quad (16)$$

where M^2 and e^2 are constants. We have retained the notation of ‘ e ’ and ‘ M ’ used in the standard GR Reissner–Nordstrom solution where they represent charge and mass, respectively. However, here, ‘ e ’ and ‘ M ’ may not carry the same physical meaning as in Reissner–Nordstrom. It is easy to check that the above-written functional forms of f and U satisfy the tracelessness criterion.

We now look at the equation for the scalar Φ . Assume $1 + \Phi = \xi^2$. The first integral of the scalar wave equation then becomes

$$\xi' = \frac{C_1}{r^2 f} = \frac{C_1}{r^2 + a^2} \quad (17)$$

where $a^2 = \frac{e^2 - M^2}{4}$ and C_1 , an integration constant. It is clear that there will be two different solutions for $a^2 > 0$ ($e^2 > M^2$) and $a^2 < 0$ ($e^2 < M^2$). Both these solutions must converge to the solution for $e^2 = M^2$ which gives the extremal limit.

When $e^2 > M^2$ (i.e. the nakedly singular solution) we obtain

$$\Phi(r) = \left(\frac{C_1}{a} \tan^{-1} \frac{r}{a} + \frac{C_4}{2} \right)^2 - 1 \quad (18)$$

This solution remains valid for all $r \geq 0$ with a choice of $C_4 > 2$ necessary to ensure the positivity condition $\Phi(r) > 0$. In addition, note that the radion is finite everywhere including the asymptotic region $r \rightarrow \infty$. In the limit $a \rightarrow 0$, this solution for Φ will reduce to that for the extremal case, given as

$$\Phi(r) = \left(\frac{C_1}{r} + \frac{C_4}{2} \right)^2 - 1 \quad (19)$$

Here $C_1 > 0$ and $C_4 > 2$ is a requirement for positivity of the brane separation. Figure 1 shows the cases with $C_1 = 1$, $a = 2$, $C_4 = 3$ (blue curve) and $C_4 = -3$ (red curve). It may be noted from the figure that for $C_4 = -3$ the radion ends up having zeros and is therefore such a choice of C_4 is not permitted.

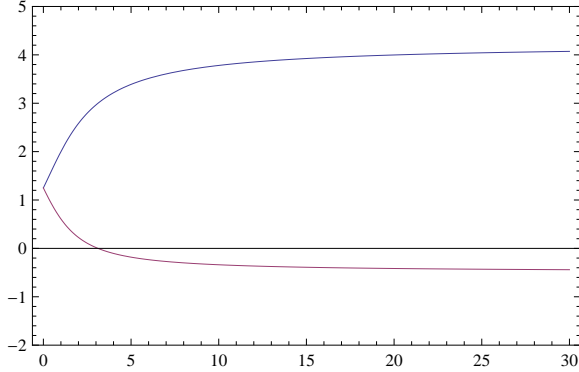


FIG. 1: $\Phi(r)$ vs. r for $a^2 > 0$; $C_1 = 1, a = 2, C_4 = 3$ (blue), $C_4 = -3$ (red)

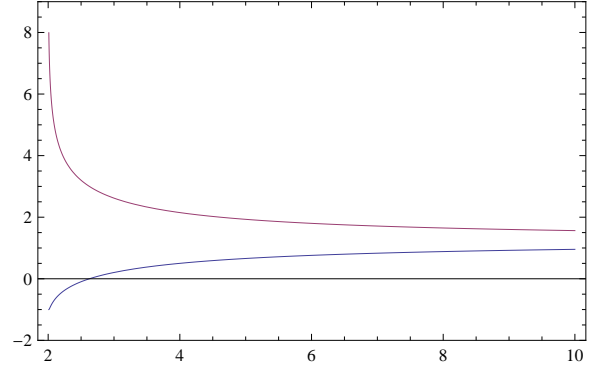


FIG. 2: $\Phi(r)$ vs. r for $a^2 < 0$; $C_1 = 1, a = 2, C_4 = -3$ (red), $C_4 = 3$ (blue)

When $e^2 < M^2$ (i.e. the black hole solution with horizon at $r = a$, $a = \sqrt{-a^2}$), we obtain

$$\Phi(r) = \left(\frac{C_1}{2a} \ln \left| \frac{r-a}{r+a} \right| + \frac{C_4}{2} \right)^2 - 1 \quad (20)$$

This solution is valid for all $r \geq a$. It may be observed that at the location of the horizon, the inter-brane distance becomes infinitely large (see Figure 2). As shown later, this divergence implies a divergent matter stress energy at the horizon, thereby making the solution physically disallowed. Note also that for $C_4 = 3$ the radion has zeros and therefore such a choice of the parameters is not permissible. In the limit $a \rightarrow 0$ (from the $a^2 < 0$ side) limit, the above solution does approach the one mentioned above for $a = 0$.

Finally, let us obtain the radion for the Schwarzschild case, i.e. when $e = 0$. This turns out to be:

$$\Phi = \left(\frac{C_1}{M} \ln \frac{2r-M}{2r+M} + C_4 \right)^2 - 1 \quad (21)$$

One can easily show from the above expression that on the horizon at $r = \frac{M}{2}$, Φ diverges making the solution again physically unacceptable. We illustrate this behaviour of radion profile in Figure 3. We note that apart from the divergence of the radion at the horizon, the radion profile hits $\Phi = 0$ at locations outside the horizon. This is a generic feature. Coupled with the radion divergence at the horizon, one is forced to conclude that a physically allowed radion is not possible in a Schwarzschild spacetime.

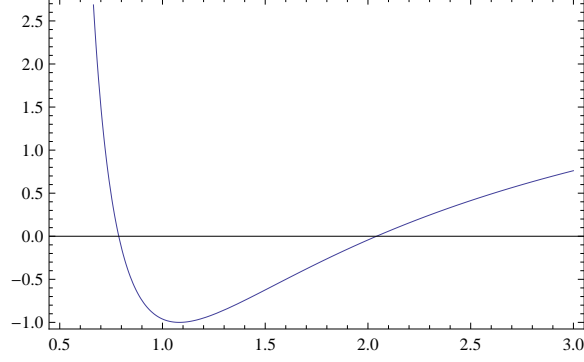


FIG. 3: $\Phi(r)$ vs. r for Schwarzschild; $C_4 = 2, C_1 = 2, M = 1$

B. The Weak Energy Condition inequalities

Using the expressions of U , Φ and f mentioned in the previous subsection, in Eqns. (9)-(11) we obtain non-zero ρ , τ and p , i.e. we have non-trivial on-brane matter.

It is now necessary to see if the matter is reasonable in terms of satisfying the energy conditions [7] and whether the $a \rightarrow 0$ limit yields our earlier result [4].

From the field equations we obtain,

$$\frac{\kappa^2}{\ell} \rho = \Phi \frac{e^2}{U^2 r^4} + \frac{\alpha^2 M^2}{r^4 f^2} - \frac{\Phi'}{U f} \left(-\frac{M}{r^2} + \frac{4a^2}{r^3} \left(1 + \frac{M}{4r} \right) \right) \quad (22)$$

$$\frac{\kappa^2}{\ell} \tau = -\Phi \frac{e^2}{U^2 r^4} + \frac{3\alpha^2 M^2}{r^4 f^2} + \Phi' \left(\frac{f}{U r} + \frac{1}{r} - \frac{2a^2}{r^3 f} \right) \quad (23)$$

where we have used $C_1 = \alpha M$ where α is a proportionality constant.

The pressure p can be obtained from the tracelessness condition. It is given as,

$$\frac{\kappa^2}{\ell} p = \frac{1}{2} (\rho - \tau) \quad (24)$$

Recall that $\rho \geq 0$, $\rho + \tau \geq 0$, $\rho + p \geq 0$ constitutes the Weak Energy Condition (WEC) and the subset $\rho + \tau \geq 0$, $\rho + p \geq 0$ defines the Null Energy Condition (NEC)) [7]. The analysis of the energy conditions will help us know about the nature of the traceless matter that must be there on the brane, if the solution has to exist.

It is useful to write down the expressions for $\rho + \tau$ and $\rho + p$ in a compact form. They are

given as:

$$\frac{\kappa^2}{\ell} (\rho + \tau) = \frac{2}{f^2 r} \left[\Phi' \left(1 - \frac{a^4}{r^4} \right) + \frac{2\alpha^2 M^2}{r^3} \right] \quad (25)$$

$$\frac{\kappa^2}{\ell} (\rho + p) = \frac{1}{r^4} \left[\frac{2\Phi e^2}{U^2} + \frac{\Phi'}{2Uf} \{r^3(U^2 - 3f^2) - ra^2\} \right] \quad (26)$$

We now analyse the various cases separately.

Case 1 ($\alpha = 1, e^2 > M^2$): In Figs. 4,5,6 we have chosen $e = 5, M = 3$ (naked singularity at $r = 1$) so that $e^2 - M^2 = 16$. We also choose $\alpha = 1$ which means that the M in the metric functions is the same as the C_1 in the radion field solution. The plot of ρ vs. r (Figure 4) demonstrates that ρ is indeed positive over the entire domain of r . Also, from the expression for ρ , it is clear that the dominant term which varies as $\frac{1}{r^2}$ as $r \rightarrow \infty$ has a positive coefficient. However, the $\rho + \tau \geq 0$ inequality is violated in a finite region around $r = 1$ (Figure 5). In the same way, we note that the $\rho + p \geq 0$ inequality is violated for large r , a fact which we demonstrate in Figure 6. In Fig. 6, the y -axis is scaled by a factor of 10^7 and we plot from $r = 100$ to $r = 1000$. The negativity of $\rho + p$ at large values of r , is evident from this figure. Thus the on-brane matter must necessarily violate the WEC if the naked singularity is a viable solution.

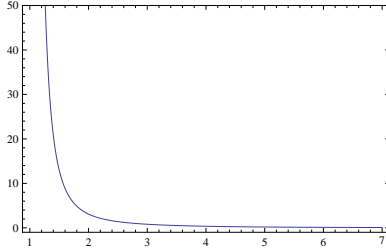


FIG. 4: $M = 3, e = 5, C_4 = 3, \alpha = 1, \rho$ vs. r .

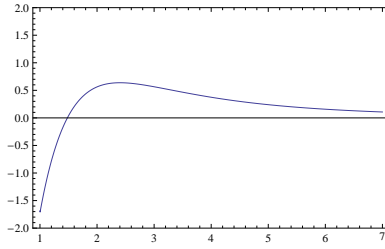


FIG. 5: $M = 3, e = 5, C_4 = 3, \alpha = 1, (\rho + \tau)$ vs. r .

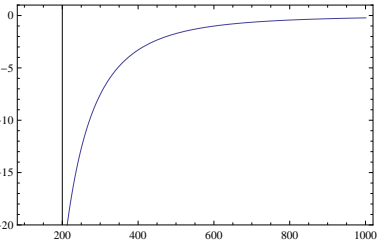


FIG. 6: $M = 3, e = 5, C_4 = 3, \alpha = 1, 10^7(\rho + p)$ vs. r .

To see this more explicitly let us go back to the compact expression for $\rho + \tau$ quoted in (25), with $\alpha = 1$. Here, note that the second term is positive but the positivity of the first term depends crucially on the sign of Φ' as well as the value of r . Now, we can have a solution Φ (for $e^2 - M^2 > 0$), with an always positive Φ' (see Figure 1, blue curve) which, in turn will ensure the positivity of the brane separation. Further, note that in this case, f is never zero. Hence the $\rho + \tau \geq 0$ inequality is never violated as long as $r > a_{crit}$. For $r < a_{crit}$ there is a violation. The value of a_{crit} is determined by the r for which the term in square

brackets in (25) turns negative. $a_{crit} < a$, as seen in Figure 5. In general, the value of a_{crit} can be found from a solution of the transcendental equation:

$$\tan^{-1} \frac{a_{crit}}{a} + \frac{C_4 a}{2M} = \frac{a a_{crit}}{a^2 - a_{crit}^2} \quad (27)$$

Further, it is easy to see that near the naked singularity one cannot avoid a violation of the WEC by any choice of the parameters. Let us evaluate the term in square brackets in (25) at $r = \frac{e-M}{2}$ (the location of the naked singularity). One finds that

$$[\dots] = \frac{16}{M(\nu - 1)^3} \left[1 - \frac{C_4}{2} - \frac{2}{\sqrt{\nu^2 - 1}} \tan^{-1} \sqrt{\frac{\nu - 1}{\nu + 1}} \right] \quad (28)$$

where $\nu = \frac{e}{M} > 1$. Note, that earlier we found $C_4 > 2$ from the requirement of positive brane separation. Thus the R. H. S. of (28) is always negative for any $\nu > 1$ and $C_4 > 2$. One can control the amount of violation by increasing M (since it appears in the denominator) such that $M^2 - e^2$ is negative. Similarly, by adjusting C_4 , M , e one can control the extent of the region (the value of a_{crit} where the WEC will be violated). But there is no way to avoid the violation of the $\rho + \tau$ inequality though it may be less in value or confined to a small region. In a similar manner, the $\rho + p$ inequality must necessarily be violated for large r . The large r limit of the $\rho + p$ expression clearly shows this feature. We have demonstrated this violation of the $\rho + p$ WEC inequality in Figure 6.

Case 2 ($e^2 - M^2 > 0, \alpha \neq 1$): In contrast, if we choose $\alpha \neq 1$ (i.e. $M \neq C_1$), the Eqn. (29) becomes

$$[\dots] = \frac{16\alpha}{M(\nu - 1)^3} \left[\alpha - \frac{C_4}{2} - \frac{2\alpha}{\sqrt{\nu^2 - 1}} \tan^{-1} \sqrt{\frac{\nu - 1}{\nu + 1}} \right] \quad (29)$$

Here, with appropriate choices of α , C_4 and ν one can satisfy the ρ , $\rho + \tau$ inequalities over the required domain, i.e. from $r = \frac{e-M}{2}$ to infinity. For example, with $M = 6$, $e = 10$, $\alpha = 6$ and $C_4 = 3$ we find that in the domain $2 \leq r \leq \infty$ there is no violation of the ρ , $\rho + \tau$ inequalities (the naked singularity is at $r = 2$). This is shown in Figures 7,8. However, the $\rho + p$ inequality still remains violated at large r , a fact we show in Figure 9.

By tuning α , C_4 and ν one can move around and reduce the extent of WEC violation though it cannot be avoided completely for the $\rho + p$ inequality. Thus, for a naked singularity, WEC violation of on-brane matter is necessary and this conforms with the Cosmic Censorship Hypothesis [7].

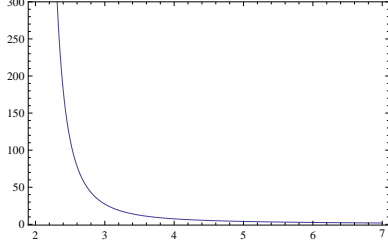


FIG. 7: $M = 6, e = 10, C_4 = 3, \alpha = 6, \rho$ vs. r .

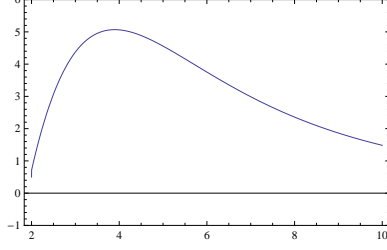


FIG. 8: $M = 6, e = 10, C_4 = 3, \alpha = 6, (\rho + \tau)$ vs. r .

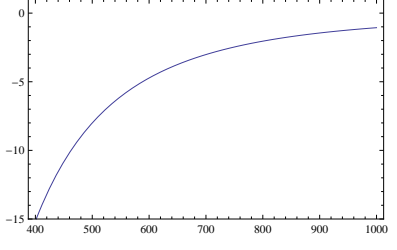


FIG. 9: $M = 6, e = 10, C_4 = 3, \alpha = 6, 10^6(\rho + p)$ vs. r .

Recall that when $e^2 - M^2 \leq 0$, the Φ diverges at the black hole horizon $r = a$ which makes the RN black hole solution unphysical. The divergence of Φ also implies a divergence of ρ , τ and p , as is evident from (22),(23). Thus, we do not discuss this case any further here.

In our earlier paper, we had noted that the extremal limit solution also does require on-brane matter which violates the energy condition inequality $\rho + \tau \geq 0$. Here we have seen that this violation persists for all $e^2 > M^2$.

IV. CONCLUSION

In this article, we have discussed the viability of the various well known GR solutions like Schwarzschild and Reissner–Nordstrom in the context of the KS theory of gravity. The crucial element in this work is related to finding a stable radion which is finite and non-zero everywhere. The RN black hole solution requires an infinite inter-brane distance at the horizon – a fact which makes it unphysical. On the other hand, the Schwarzschild solution requires a radion which diverges at the horizon, vanishes at two values of r outside the horizon and is negative between these values. The RN naked singularity and the extremal RN solution do have a non-zero and finite radion, but our analysis shows that the required on-brane matter violates the Weak and the Null Energy Condition. For the naked singularity, this feature conforms with the Cosmic Censorship Conjecture [7].

A possible way out of the problems mentioned here is to look for solutions which are non-singular in nature and see if the radion is finite and non-zero everywhere and the on-brane matter satisfies the energy conditions. The finiteness of the radion seems to be in conflict with the existence of a black hole horizon. Does this mean that there are no eternal black hole solutions in this theory? We have not proved any such statement but the analysis on

the RN and Schwarzschild solutions seem to suggest such an outcome.

Finally, it may be useful to assume a specific form of the on-brane matter (on either or both branes), and then find the radion and the metric functions. This will genuinely be like *finding an exact solution*, given the matter content on the branes. However, knowing the complicated nature of the field equations, this will not be easy to do. Further, if we remove the traceless requirement, the equations will become even more difficult to solve.

One might view the KS theory as a scalar-tensor theory in its own right. Then, of course, the radion is just another scalar field without any reference to braneworlds and it need not satisfy the requirements we have mentioned in this paper. However, such an approach is not the main motivation of this article where we have chosen to view the radion as related to the proper distance between branes located in a higher dimensional bulk spacetime.

-
- [1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); *ibid* **83**, 4690 (1999).
 - [2] T. Shiromizu, K. Maeda and M. Sasaki, Phys. Rev. **D62**, 024012 (2000), R. Maartens, Brane world gravity, Living Rev. Relativity **13**, 5 (2010).
 - [3] S. Kanno, J. Soda, Phys.Rev. **D 66**, 083506 (2002); T. Shiromizu and K. Koyama, Phys. Rev. **D 67**, 104011 (2003)
 - [4] S. Kar, S. Lahiri and S. SenGupta, Phys. Rev. **D 88** 123509, (2013) and references therein.
 - [5] S. D. Majumdar, Phys. Rev. **72**, 390 (1947); A. Papapetrou, Proc. Royal Irish Academy **A 51** 191 (1947).
 - [6] N. Dadhich, R. Maartens, P. Papadopoulos, V. Rezanian, Phys. Letts. **B487**,1 (2000)
 - [7] R. M. Wald, General Relativity, University of Chicago Press, First Indian Edition (2006).